OCR
Oxford Cambridge and RSA

Wednesday 13 May 2015 – Morning

AS GCE MATHEMATICS

4721/01 Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4721/01
- List of Formulae (MF1)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

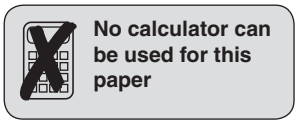
These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



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1 Express $\frac{8}{\sqrt{3}-1}$ in the form $a\sqrt{3}+b$, where a and b are integers. **[3]**

2 **(i)** Sketch the curve $y=-\frac{1}{x}$. **[2]**

(ii) The curve $y=-\frac{1}{x}$ is translated by 2 units parallel to the x-axis in the positive direction. State the equation of the transformed curve. **[2]**

(iii) Describe a transformation that transforms the curve $y=-\frac{1}{x}$ to the curve $y=-\frac{1}{3x}$. **[2]**

3 Express each of the following in the form 5^k.

(i) 25^4 **[1]**

(ii) $\frac{1}{\sqrt[4]{5}}$ **[2]**

(iii) $(5\sqrt{5})^3$ **[2]**

4 Solve the equation $x^{\frac{2}{3}}-x^{\frac{1}{3}}-6=0$. **[5]**

5 The points A and B have coordinates (2, 1) and (5, –3) respectively.

(i) Find the length of AB. **[2]**

(ii) Find an equation of the line through the mid-point of AB which is perpendicular to AB, giving your answer in the form $ax+by+c=0$ where a, b and c are integers. **[7]**

6 Solve the simultaneous equations

$$2x+y-5=0, \qquad x^2-y^2=3. \quad \textbf{[5]}$$

7 **(a)** Given that $\mathrm{f}(x)=(x^2+3)(5-x)$, find $\mathrm{f}'(x)$. **[4]**

(b) Find the gradient of the curve $y=x^{-\frac{1}{3}}$ at the point where $x=-8$. **[4]**

8 **(i)** Sketch the curve $y = 2x^2 - x - 3$, giving the coordinates of all points of intersection with the axes. **[4]**

(ii) Hence, or otherwise, solve the inequality $2x^2 - x - 3 > 0$. **[2]**

(iii) Given that the equation $2x^2 - x - 3 = k$ has no real roots, find the set of possible values of the constant k. **[3]**

9 The curve $y = 2x^3 - ax^2 + 8x + 2$ passes through the point B where $x = 4$.

(i) Given that B is a stationary point of the curve, find the value of the constant a. **[5]**

(ii) Determine whether the stationary point B is a maximum point or a minimum point. **[2]**

(iii) Find the x-coordinate of the other stationary point of the curve. **[3]**

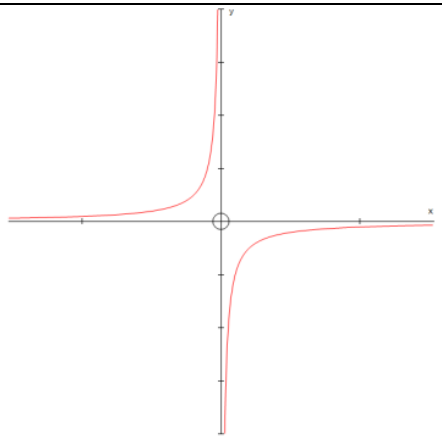
10 A circle with centre C has equation $x^2 + y^2 - 10x + 4y + 4 = 0$.

(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Show that the tangent to the circle at the point P (8, 2) has equation $3x + 4y = 32$. **[5]**

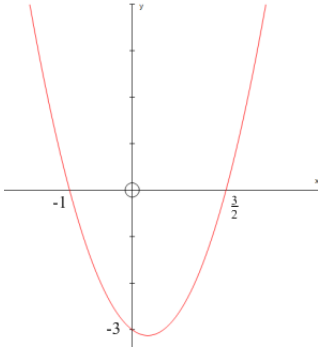
(iii) The circle meets the y-axis at Q and the tangent meets the y-axis at R. Find the area of triangle PQR. **[4]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$\frac{8}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	M1 A1 A1 **[3]**	Multiply top and bottom by $\sqrt{3}+1$ or $-\sqrt{3}-1$ – evidence of multiplying out needed Either numerator or denominator correct Final answer **cao**	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{3}+b$ **A1** Either a or b correct **A1** Both correct
2	(i)			B2 **[2]**	Excellent curve in both quadrants: • correct shape, symmetrical, not touching axes • asymptotes clearly the axes • not finite • allow slight movement away from asymptote at one end but not more.	**N.B. Ignore 'feathering' now that answers are scanned.** **B1** only – correct shape in 2nd and 4th quadrants **only**. Graph must not touch axes more than once. Finite "plotting" condoned.
	(ii)		$y=-\frac{1}{x-2}$ oe	M1 A1 **[2]**	$(y=)-\frac{1}{x-2}$ or $(y=)-\frac{1}{x+2}$ Fully correct, must include "$y=$"	$(y=)\frac{1}{x+2}$ or $(y=)\frac{1}{x-2}$ is **M0**
	(iii)		Stretch Scale factor $\frac{1}{3}$ parallel to the x-axis (or y-axis)	B1 B1 **[2]**	Stretch or "stretched" etc.; **do not accept** squashed, compressed, enlarged etc. Correct description Condone just "factor $\frac{1}{3}$" but **no reference to units.** Must not follow e.g. "reflection"	**0/2** if more than one type of transformation mentioned **ISW** non-contradictory statements For "parallel to the x/y axis" allow "vertically", "in the x/y direction". **Do not accept** "in/on/ across/up/along/to/towards the x/y axis"

Question			Answer	Marks	Guidance	
3	(i)		5^8	B1 **[1]**	**cao**	
	(ii)		$5^{-\frac{1}{4}}$	M1 A1 **[2]**	Fourth root **≡** $\frac{1}{4}$ soi **cao www**	
	(iii)		$5^{\frac{9}{2}}$	M1 A1 **[2]**	$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5 oe **cao www**	
4			$k = x^{\frac{1}{3}}$ $k^2 - k - 6 = 0$ $(k-3)(k+2) = 0$ $k = 3, k = -2$ $x = 3^3, x = -2^3$ $x = 27, x = -8$	M1* M1dep A1 M1 A1 **[5]**	Use a substitution to obtain a quadratic, or factorise into 2 brackets each containing $x^{\frac{1}{3}}$ Attempt to solve resulting three-term quadratic – **see guidance in appendix 1** Correct values of k Attempt to cube at least one value Final answers correct **ISW**	**No marks** if whole equation cubed/ rooted etc. **No marks** if straight to quadratic formula with no evidence of substitution at start and no cube rooting/cubing at end. **Spotted solutions:** If **M0 DMO** or **M1 DM0** **SC B1** $x = 27$ **www** **SC B1** $x = -8$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)
5	(i)		$AB = \sqrt{(5-2)^2 + (-3-1)^2}$ $AB = 5$	M1 A1 **[2]**	Attempt to use Pythagoras' theorem – 3/4 numbers substituted correctly **and attempt to square root** Final answer correct, must be fully processed. ±5 is A0.	

Question			Answer	Marks	Guidance	
	(ii)		$\left(\frac{2+5}{2}, \frac{1+-3}{2}\right)$	M1	Correct method to find mid-point of line	Alternative using general point on the perpendicular
			$(3.5, -1)$	A1		**M2** States P (x, y) a point on the perpendicular and attempts PA = PB or $PA^2 = PB^2$
			Gradient of AB = $-\frac{4}{3}$	B1	Processed	**A1** At least one of PA, PB correct
			Perpendicular gradient = $\frac{3}{4}$	B1**ft**	$\frac{-1}{\text{their gradient}}$ processed	**A1** Both correct **M1** Expands and simplifies
			$y+1=\frac{3}{4}(x-\frac{7}{2})$	M1	Equation of straight line through their mid-point, any non-zero gradient in any form	**A1** Correct equation found
				A1		**A1** Correct equation in required form
			$6x-8y-29=0$	A1	**cao** Must be correct equation in required form i.e. $k(6x-8y-29)=0$ for integer k.	
				[7]	**Must have "=0"**	
6			$x^2-(5-2x)^2=3$	M1*	Substitute for x/y or valid attempt to eliminate one of the variables	If y eliminated: $3y^2+10y-13=0$
			$3x^2-20x+28=0$	A1	Three term quadratic in solvable form	$(3y+13)(x-1)=0$
			$(3x-14)(x-2)=0$	M1dep	Correct method to solve three term quadratic – **see appendix 1**	**Spotted solutions: If M*0** **SC B1** $x=2, y=1$ **www**
			$x=\frac{14}{3}, x=2$	A1	Both x values correct	**SC B1** $x=\frac{14}{3}, y=-\frac{13}{3}$ **www** Must show on both line and curve
			$y=-\frac{13}{3}, y=1$	A1 **[5]**	Both y values correct. **Allow 1 A mark for one correct pair of x and y from correct factorisation.**	(Can then get 5/5 if both found **www** and exactly two solutions justified)
7	**(a)**		$(x^2+3)(5-x)=5x^2-x^3+15-3x$	M1	Attempt to multiply out brackets, Must have four terms, at least three correct	<u>Alternative using product rule:</u> Clear attempt at correct rule **M1***
				A1	Fully correct expression. Do not ISW if signs then changed. Max 2/4.	Both expressions fully correct **A1** Expand brackets of both parts
			$\frac{dy}{dx} = 10x-3x^2-3$	M1	Attempt to differentiate their expression, (power of at least one term involving x reduced by one)	**M1*dep** Fully correct expression **A1**
				A1 **[4]**		

Question			Answer	Marks	Guidance	
	(b)		$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}}$	M1	Attempt to differentiate i.e. $-\frac{1}{3}x^{-\frac{k}{3}}$ soi for positive integer k	$x^{-\frac{1}{3}}$ misread as $x^{\frac{1}{3}}$ earns max 2/4: $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ **M1 A0 MR**
			When $x = -8$ $\frac{dy}{dx} = -\frac{1}{3} \times (-8)^{-\frac{4}{3}}$	A1 B1	Fully correct $(-8)^{-\frac{4}{3}} = \frac{1}{16}$ **www** Must use – 8	$(-8)^{-\frac{2}{3}} = \frac{1}{4}$ **B1**
			$\frac{dy}{dx} = -\frac{1}{3} \times \frac{1}{16} = -\frac{1}{48}$	A1 **[4]**	Final answer	Final answer $\frac{1}{12}$ **A0 MR**
8	**(i)**		$(2x-3)(x+1) = 0$ $x = \frac{3}{2}, x = -1$ 	M1 A1 A1**ft** B1 **[4]**	Correct method to find roots – **see appendix 1** Correct roots Good curve: • Correct shape, symmetrical positive quadratic • Minimum point in the correct quadrant for their roots **(ft)** • their x intercepts correctly labelled **(ft)** y intercept at (0, −3). Must have a graph.	
8	**(ii)**		$x < -1, x > \frac{3}{2}$	M1 A1**ft** **[2]**	Chooses the "outside region" Follow through x-values in (i). Allow "$x < -1, x > \frac{3}{2}$", "$x < -1$ **or** $x > \frac{3}{2}$" but do not allow "$x < -1$ **and** $x > \frac{3}{2}$"	If restarted, fully correct method for solving a quadratic inequality including choosing "outside region" needed for **M1** **NB** e.g. $-1 > x > \frac{3}{2}$ scores **M1A0** Must be strict inequalities for **A** mark

Question			Answer	Marks	Guidance	
8	(iii)		$b^2 - 4ac = 1^2 - 4 \times 2 \times -(3 + k)$	M1	Rearrangement and use of $b^2 - 4ac < 0$, must involve 3 and k in constant term (not $3k$)	Alt for first two marks: **M1** Attempt to find turning point and form inequality $k < y_{min}$
			$25 + 8k < 0$	A1	$p + 8k < 0$ oe found, any constant p. p need not be simplified	**A1** turning point correct $(\frac{1}{4}, -\frac{25}{8})$
			$k < -\frac{25}{8}$	A1 **[3]**	Correct final answer	**If M0** (either scheme) **SC B1** $k = -\frac{25}{8}$ or $k > -\frac{25}{8}$ seen
9	(i)		$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1	Attempt to differentiate, at least two non-zero terms correct	
				A1	Fully correct	
			When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	These Ms may be awarded in either order
			$\frac{dy}{dx} = 0$ gives $a = 13$	M1 A1 **[5]**	Sets their $\frac{dy}{dx}$ to 0. Must be seen	
	(ii)		$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign	**Alternate valid methods include:** 1) Evaluating gradient at either side of 4($x > \frac{1}{3}$) e.g. at 3, –16 at 5, 28 2) Evaluating $y = -46$ at 4 and either side of 4 ($x > \frac{1}{3}$) e.g. (3, – 37), (5, – 33)
			When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	A1 **[2]**	**www**	If using alternatives, working must be fully correct to obtain the **A** mark
	(iii)		$6x^2 - 26x + 8 = 0$	M1	Sets their derivative to zero	
			$(3x - 1)(x - 4) = 0$	M1	Correct method to solve quadratic **(appx 1)**	Could be $(6x - 2)(x - 4) = 0$ or $(3x - 1)(2x - 8) = 0$
			$x = \frac{1}{3}$	A1 **[3]**	oe	

Question			Answer	Marks	Guidance	
10	(i)		$C = (5, -2)$	B1	Correct centre	
			$(x-5)^2 + (y+2)^2 - 25 = 0$	M1	$(x \pm 5)^2 - 5^2$ and $(y \pm 2)^2 - 2^2$ seen (or implied by correct answer)	Or attempt at $r^2 = f^2 + g^2 - c$
			Radius = 5	A1	Correct radius – do not allow **A** mark from $(x+5)^2$ and/or $(y-2)^2$	± 5 or $\sqrt{25}$ A0.
				[3]		
10	(ii)		Gradient $PC = \frac{2--2}{8-5} = \frac{4}{3}$	M1	Attempt to find gradient of radius (3/4 correct)	**See also alternative methods on next page**
				A1		
			Gradient of tangent $= -\frac{3}{4}$	B1**ft**	$\frac{-1}{\text{their gradient}}$ processed	
			$y - 2 = -\frac{3}{4}(x-8)$	M1	Equation of straight line through P, using their perpendicular gradient (not from rearrangement)	Do not allow use of gradient of radius instead of tangent
			$4y + 3x = 32$	A1	Rearrange to required form **www AG**	Ignore order of terms
				[5]		
PLEASE SEE NEXT PAGE FOR 10ii ALTERNATIVE METHODS						
	(iii)		$Q = (0, -2)$	B1	Q found correctly	For the M mark, allow splitting into two triangles $\frac{1}{2} \times 6 \times 8 + \frac{1}{2} \times 4 \times 8$
			$R = (0, 8)$	B1	R found correctly	
			Area $= \frac{1}{2} \times (8 - -2) \times 8$	M1	Attempt to find area of triangle with their Q, R and height 8 i.e. $\frac{1}{2} \times (y_R - y_Q) \times 8$	If using PQ as base then expect to see $\frac{1}{2} \times \sqrt{80} \times \sqrt{80}$ **www**
			40	A1		
				[4]		

Alternative methods for 10(ii)		
Alternative by rearrangement	**Alternative for equating given line to circle**	**Alternative for implicit differentiation:**
Gradient of radius = $\frac{2--2}{8-5}=\frac{4}{3}$ **M1A1** Attempts to rearrange equation of line to find gradient of line = $-\frac{3}{4}$ and compares with gradient of radius **M1** Multiply gradients to get –1 **B1** Check (8, 2) lies on line **B1**	Substitute for *x*/*y* or attempt to get an equation in 1 variable only **M1** $k(x^2 - 16x + 64) = 0$ or $k(y^2 - 4y + 4) = 0$ **A1** Correct method to solve quadratic – **see appendix 1** **M1** $x = 8$, $y = 2$ found **A1** States one root implies tangent **B1**	**M*1** Attempt at implicit differentiation as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$ **A1** Substitution of (8, 2) to obtain – $\frac{3}{4}$ Then as main scheme **OR** Attempts to rearrange equation of line to find gradient of line = $-\frac{3}{4}$ **M1dep** Check (8, 2) lies on line **B1**

APPENDIX 1

Solving a quadratic

This is particularly important to mark correctly as it features several times on the paper.
Consider the equation: $3x^2 - 13x - 10 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x + 5)(x - 2)$	**M1** $3x^2$ and -10 obtained from expansion
$(3x - 4)(x - 3)$	**M1** $3x^2$ and $-13x$ obtained from expansion
$(3x + 5)(x + 2)$	**M0** only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\frac{-13 \pm \sqrt{(-13)^2 - 4\times3\times-10}}{2\times3}$	earns **M1** (minus sign incorrect at start of formula)
$\frac{13 \pm \sqrt{(-13)^2 - 4\times3\times10}}{2\times3}$	earns **M1** (10 for c instead of -10 is the only sign slip)
$\frac{-13 \pm \sqrt{(-13)^2 - 4\times3\times10}}{2\times3}$	**M0** (2 sign errors: initial sign and c incorrect)
$\frac{13 \pm \sqrt{(-13)^2 - 4\times3\times-10}}{2\times-10}$	**M0** ($2c$ on the denominator instead of $2a$)

Notes – for equations such as $3x^2 - 13x - 10 = 0$, then $b^2 = 13^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$3x^2 - 13x - 10 = 0$$
$$3\left(x^2 - \frac{13}{3}x\right) - 10 = 0$$
$$3\left[\left(x - \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10 = 0$$
$$\left(x - \frac{13}{6}\right)^2 = \frac{289}{36}$$
$$x - \frac{13}{6} = \pm\sqrt{\frac{289}{36}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{13}{6}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.